Adjusted EBITDA as an Inference Problem: Evidence-Weighted Addbacks and the Price of Being Wrong

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Abstract

Adjusted EBITDA is usually presented as a number. I argue that it is an estimate. Each addback in a Quality of Earnings bridge is a claim that is true with some probability given the evidence behind it, and because enterprise value is a multiple of adjusted EBITDA, every wrongly accepted dollar of addback is paid for several times over. I treat the bridge as a problem of statistical inference and derive the rules that follow. The mean and variance of adjusted EBITDA follow from the posterior validity probabilities, with a sharp bound under common-cause dependence. The valuation error of any accept or reject decision is an exact linear identity in the multiple. The Bayes-optimal acceptance rule is a probability threshold set by the ratio of error costs, independent of the size of the item, and a buyer and a seller rationally hold different thresholds. An expected-value bridge minimises mean squared error and generally differs from the accept or reject bridge. Hoeffding's inequality gives a distribution-free confidence band, vacuous for short bridges. A materiality floor bounds each omitted item but not their sum, so the count and total of exclusions must be reported. A bridge is additive only if no source row with a non-zero amount is claimed twice. Finally, a fixed review budget is best spent on the items with the largest $a_i^2 p_i(1-p_i)$: large and genuinely uncertain. All numerical examples are illustrative, and the model does not say where the probabilities come from.

1 Introduction

A private company is rarely sold on its reported earnings. It is sold on *adjusted* EBITDA: reported EBITDA plus a list of addbacks, each asserting that some historical cost was non-recurring, non-operating, above market, or otherwise unrepresentative of the business a buyer will own. The list is called the bridge. The buyer's Quality of Earnings (QoE) analysis exists to test it, and the price is then negotiated, in the simplest framing, as a multiple of the adjusted figure.

The standard presentation of a bridge is a column of amounts that sums to a single number. That presentation hides the nature of the object. An addback is not a ledger fact. The ledger records that a legal invoice was paid; it does not record that the dispute behind it will never recur. Whether the addback is *valid* is a claim about the world, supported by evidence of varying strength: a signed settlement agreement, a compensation study, a

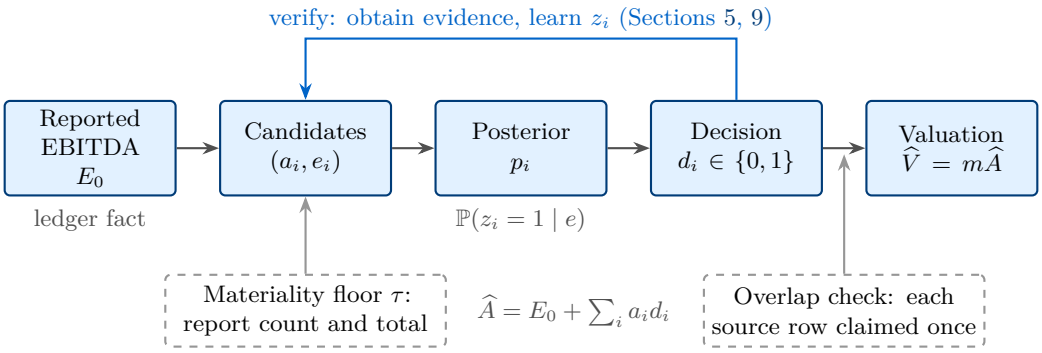


Figure 1: The bridge as an inference pipeline. Reported EBITDA is a deterministic ledger quantity. Each candidate carries an amount a_i and evidence e_i, from which a posterior validity probability p_i is formed and mapped to accept, reject, or verify. The dashed guards are the materiality floor (Section 7) and the overlap check that makes the sum meaningful (Section 8).

management representation, or nothing at all. A sum of claims that are each true only with some probability is a random variable, and the figure on the page is one summary of its distribution, chosen implicitly.

Two strands of prior work motivate making this explicit. The first is information asymmetry. Akerlof [1970] showed that when sellers know quality and buyers do not, the buyer must price on beliefs, and the market can unravel. A sell-side bridge is exactly a quality claim made by the better-informed party. The second is the accounting literature on earnings quality. Sloan [1996] showed that the accrual component of earnings is less persistent than the cash component; Dechow and Dichev [2002] measure accrual quality by how well working-capital accruals map into realised cash flows, treating accruals as estimates with error; Richardson et al. [2005] tie lower accrual reliability to lower earnings persistence; Jones [1991] models expected accruals in order to isolate the discretionary part; Beneish [1999] builds a statistical screen for earnings manipulation from financial-statement ratios; and Dechow et al. [2010] review the proxies and stress that earnings quality is defined relative to a decision. All of these treat reported earnings as a noisy measurement. I apply the same stance one level down, to the individual adjustments of a bridge.

The contribution is a small, exact theory: moments and dependence (Section 3), the price of an acceptance error (Section 4), optimal acceptance (Section 5), confidence bands (Section 6), the materiality floor (Section 7), double counting (Section 8), where to spend review effort (Section 9), and an illustrative bridge (Section 10).

The paper contains no empirical results. It does not say what the probability of a given addback is; it says what follows once such probabilities exist.

2 Setup

Definition 2.1 (Bridge)**.** A *bridge* consists of a reported EBITDA $E_0 \in \mathbb{R}$ and n candidate adjustments. Adjustment i has a signed amount $a_i \in \mathbb{R} \setminus \{0\}$ (positive for an addback that raises EBITDA, negative for a deduction), an unobserved validity indicator $z_i \in \{0, 1\}$, and

evidence e_i. The *true adjusted EBITDA* is

$$A \;=\; E_0 + \sum_{i=1}^{n} a_i z_i. \tag{1}$$

The amounts a_i and E_0 are taken as known: they are sums of ledger rows. All uncertainty sits in $z = (z_1, \ldots, z_n)$. Write $e = (e_1, \ldots, e_n)$ for the whole evidence record. The analyst's state of knowledge is a posterior distribution $\mathbb{P}(\cdot \mid e)$ on $\{0,1\}^n$, and every probability, expectation and variance below is taken under it. Its marginals are

$$p_i \;=\; \mathbb{P}(z_i = 1 \mid e), \qquad q_i = 1 - p_i. \tag{2}$$

If the evidence for other items is uninformative about item i once e_i is known, then $p_i = \mathbb{P}(z_i = 1 \mid e_i)$, which is the form in which a reviewer usually thinks. I say the adjustments are *independent* when $z_1, \ldots, z_n$ are mutually independent under the posterior.

Definition 2.2 (Valuation)**.** For a multiple $m > 0$, the enterprise value implied by adjusted EBITDA is $V = mA$. A *decision* is a vector $d = (d_1, \ldots, d_n) \in \{0,1\}^n$ that may depend on e but not on z; $d_i = 1$ means adjustment i is accepted. The *decided bridge* and the price it implies are

$$\widehat{A} = E_0 + \sum_{i=1}^{n} a_i d_i, \qquad \widehat{V} = m\widehat{A}. \tag{3}$$

The multiple is treated as a known constant, and validity as binary; both simplifications are revisited in Section 13.

3 Moments of Adjusted EBITDA

Let $v_i = a_i^2 p_i q_i$ and $s_i = |a_i|\sqrt{p_i q_i}$, the variance and standard deviation of the single term $a_i z_i$.

Proposition 3.1 (Mean and variance)**.** *For any posterior,*

$$\mathbb{E}[A] = E_0 + \sum_{i=1}^{n} a_i p_i, \qquad \operatorname{Var}(A) = \sum_{i=1}^{n} v_i + \sum_{i \neq j} a_i a_j \operatorname{Cov}(z_i, z_j). \tag{4}$$

If the z_i are pairwise uncorrelated, in particular if they are independent, the second sum vanishes and $\operatorname{Var}(A) = \sum_i a_i^2 p_i (1 - p_i)$.

Proof. The mean follows from linearity and $\mathbb{E}[z_i] = p_i$. The variance of a sum is the sum of all pairwise covariances, $\sum_{i,j} a_i a_j \operatorname{Cov}(z_i, z_j)$, and $z_i^2 = z_i$ gives $\operatorname{Var}(z_i) = p_i - p_i^2 = p_i q_i$. The cross terms vanish when the z_i are pairwise uncorrelated. □

The independent formula is not safe by default. The next result bounds the standard deviation with no assumption on dependence at all.

Proposition 3.2 (Dependence-free bound)**.** *For any posterior,*

$$\mathrm{sd}(A) \;\le\; \sum_{i=1}^{n} |a_i| \sqrt{p_i(1-p_i)} \;=\; \sum_{i=1}^{n} s_i. \tag{5}$$

Equality holds if and only if, for every pair $i \neq j$ with $p_i, p_j \in (0,1)$, either $a_i a_j > 0$ and $z_i = z_j$ almost surely, or $a_i a_j < 0$ and $z_i = 1 - z_j$ almost surely.

Proof. Let $X_i = a_i z_i$, so $\mathrm{sd}(X_i) = s_i$. By the Cauchy-Schwarz inequality, $\mathrm{Cov}(X_i, X_j) \le s_i s_j$, hence

$$\mathrm{Var}(A) = \sum_{i,j} \mathrm{Cov}(X_i, X_j) \;\le\; \sum_{i,j} s_i s_j = \Big(\sum_i s_i \Big)^2,$$

which is the triangle inequality for the standard deviation. Equality holds if and only if $\mathrm{Cov}(X_i, X_j) = s_i s_j$ for every pair. Pairs with $s_i s_j = 0$ satisfy this automatically. For a pair with $s_i s_j > 0$, equality in Cauchy-Schwarz means $X_j - \mathbb{E}X_j = \lambda(X_i - \mathbb{E}X_i)$ almost surely for some $\lambda > 0$, that is, $z_j - p_j = \lambda(a_i/a_j)(z_i - p_i)$. Both z_i and z_j take exactly the values 0 and 1 with positive probability. If $a_i a_j > 0$, then z_j is an increasing affine function of z_i, which forces $z_j = z_i$. If $a_i a_j < 0$, it is a decreasing affine function, which forces $z_j = 1 - z_i$. Conversely, each of these relations gives correlation $+1$ between X_i and X_j. □

The equality case is restrictive: $z_i = z_j$ almost surely forces $p_i = p_j$, so (5) is never attained by same-signed addbacks with different probabilities. The exact worst case is the following.

Proposition 3.3 (Sharp bound for same-signed adjustments)**.** *Suppose all a_i have the same sign. Then, over all posteriors with marginals $p_1, \ldots, p_n$,*

$$\max \mathrm{Var}(A) \;=\; \sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j \big(\min(p_i, p_j) - p_i p_j \big), \tag{6}$$

attained by the comonotone coupling $z_i = \mathbf{1}\{U \le p_i\}$ with U uniform on $[0,1]$.

Proof. For any joint law, $\mathbb{P}(z_i = 1, z_j = 1) \le \min(p_i, p_j)$, so $\mathrm{Cov}(z_i, z_j) \le \min(p_i, p_j) - p_i p_j$, and for $i = j$ this is the identity $\mathrm{Var}(z_i) = p_i - p_i^2$. Since $a_i a_j > 0$ for every pair, $\mathrm{Var}(A) = \sum_{i,j} a_i a_j \,\mathrm{Cov}(z_i, z_j)$ is at most the right side of (6). Under the comonotone coupling, $\mathbb{P}(z_i = 1, z_j = 1) = \mathbb{P}(U \le \min(p_i, p_j)) = \min(p_i, p_j)$ for all pairs simultaneously, so the bound is attained. □

Remark 3.4 (Why common cause is the realistic danger)**.** The comonotone coupling has a plain reading: there is a single latent degree of credibility U, and every addback whose probability exceeds it stands. That is what one management narrative behind several addbacks looks like. If a group G of adjustments rests on one representation with probability p, it contributes $(\sum_{i \in G} a_i)^2 p(1-p)$ to the variance instead of $\sum_{i \in G} a_i^2 \, p(1-p)$; for k equal same-signed items the first is k times the second. Against a common cause, independence is the optimistic assumption for a bridge of same-signed addbacks, and (6) is the pessimistic one.

4 The Price of Error

Theorem 4.1 (Valuation error identity)**.** *For any decision d and any realisation z,*

$$\widehat{V} - V \;=\; m\sum_{i=1}^{n} a_i(d_i - z_i) \;=\; m\Big(\sum_{i\in \mathrm{FA}} a_i \;-\; \sum_{i\in \mathrm{FR}} a_i\Big), \tag{7}$$

where $\mathrm{FA} = \{i : d_i = 1, z_i = 0\}$ *is the set of false acceptances and* $\mathrm{FR} = \{i : d_i = 0, z_i = 1\}$ *the set of false rejections. In particular, if the only error is a wrongly accepted addback of amount $a > 0$, the buyer overpays by exactly ma; if the only error is a wrongly rejected addback of amount $a > 0$, the seller is underpaid by exactly ma. For a deduction ($a < 0$) the roles are exchanged. Under the posterior,*

$$\mathbb{E}[\widehat{V} - V] = m\sum_{i=1}^{n} a_i(d_i - p_i), \qquad \mathbb{E}[(\widehat{V} - V)^2] = m^2 \operatorname{Var}(A) + m^2\Big(\sum_{i=1}^{n} a_i(d_i - p_i)\Big)^2. \tag{8}$$

Proof. Subtract (1) from (3) and multiply by m. The term $d_i - z_i$ equals $+1$ on FA, -1 on FR, and 0 otherwise, which gives the second form. The special cases are the instances in which exactly one term is non-zero. The first identity in (8) is linearity. For the second, $\widehat{A}$ is a constant under the posterior, so $\mathbb{E}[(\widehat{A} - A)^2] = \operatorname{Var}(A) + (\widehat{A} - \mathbb{E}[A])^2$, and $\widehat{A} - \mathbb{E}[A] = \sum_i a_i(d_i - p_i)$. □

The identity is trivial as algebra and important as economics: the sensitivity of price to one acceptance decision is ma_i, not a_i. A reviewer deciding whether an addback of 300 stands is, at a multiple of six, deciding who bears 1,800. Errors of opposite sign net in $\widehat{V} - V$, so a bridge can be right in total and wrong in every line; the item-level loss of Section 5 does not allow that netting.

5 Optimal Acceptance

The two kinds of error are the two error types of hypothesis testing [Neyman and Pearson, 1933], and here they have prices. Let $c_{\mathrm{FA}} > 0$ be the loss per dollar of a falsely accepted adjustment and $c_{\mathrm{FR}} > 0$ the loss per dollar of a falsely rejected one. The loss of a decision d when the truth is z is

$$L(d, z) \;=\; \sum_{i=1}^{n} |a_i|\big(c_{\mathrm{FA}}\, d_i(1 - z_i) + c_{\mathrm{FR}}\,(1 - d_i)z_i\big). \tag{9}$$

Theorem 5.1 (Bayes-optimal acceptance)**.** *For any posterior, with or without independence, a decision d minimises the posterior expected loss $\mathbb{E}[L(d, z)]$ if and only if, for every i,*

$$d_i = 1 \;\; \textit{when} \;\; p_i > \theta, \qquad d_i = 0 \;\; \textit{when} \;\; p_i < \theta, \qquad \theta \;=\; \frac{c_{\mathrm{FA}}}{c_{\mathrm{FA}} + c_{\mathrm{FR}}}, \tag{10}$$

with either choice optimal when $p_i = \theta$. In particular the rule "accept if and only if $p_i \geq \theta$" is optimal, the threshold does not depend on a_i, and under symmetric loss ($c_{\mathrm{FA}} = c_{\mathrm{FR}}$) it equals $\frac{1}{2}$. The minimal expected loss is $\sum_i |a_i| \min\big(c_{\mathrm{FA}}(1 - p_i),\, c_{\mathrm{FR}}\, p_i\big)$.

Proof. By linearity of expectation,

$$\mathbb{E}[L(d,z)] = \sum_{i=1}^{n} |a_i|(c_{\mathrm{FA}}\, d_i(1-p_i) + c_{\mathrm{FR}}\,(1-d_i)\,p_i).$$

The ith summand depends on d only through d_i, so the sum is minimised by minimising each summand separately. Since $|a_i| > 0$, the choice $d_i = 1$ is strictly better than $d_i = 0$ exactly when $c_{\mathrm{FA}}(1-p_i) < c_{\mathrm{FR}}\,p_i$, that is, when $p_i > c_{\mathrm{FA}}/(c_{\mathrm{FA}} + c_{\mathrm{FR}})$; it is strictly worse when the inequality is reversed, and the two are equal at $p_i = \theta$. Substituting the smaller value in each summand gives the minimal loss. No joint property of the z_i was used. □

Size does not change *whether* to accept at a given probability, because the loss is proportional to $|a_i|$; it changes how much it is worth spending to move that probability.

Proposition 5.2 (Three-way rule with a verification option)**.** *Suppose that, in addition to accepting or rejecting, the reviewer may verify item i at cost $\kappa_i \geq 0$ (in loss units), after which z_i is known and the item is decided without error. Then the expected loss for item i is minimised by verifying if and only if*

$$\kappa_i \;<\; |a_i|\min\left(c_{\mathrm{FA}}(1-p_i),\, c_{\mathrm{FR}}\,p_i\right), \quad \textit{equivalently} \quad \frac{\kappa_i}{c_{\mathrm{FR}}|a_i|} \;<\; p_i \;<\; 1 - \frac{\kappa_i}{c_{\mathrm{FA}}|a_i|}, \tag{11}$$

and otherwise by the threshold rule of Theorem 5.1 (with indifference at equality). The verification interval is non-empty if and only if $|a_i| > \kappa_i(c_{\mathrm{FA}} + c_{\mathrm{FR}})/(c_{\mathrm{FA}}c_{\mathrm{FR}})$.

Proof. The three actions have expected losses $|a_i|c_{\mathrm{FA}}(1-p_i)$ (accept), $|a_i|c_{\mathrm{FR}}\,p_i$ (reject) and κ_i (verify). Verification is strictly best exactly when κ_i is below the smaller of the other two, which is the first form. The minimum exceeds $\kappa_i/|a_i|$ exactly when both $c_{\mathrm{FR}}\,p_i > \kappa_i/|a_i|$ and $c_{\mathrm{FA}}(1-p_i) > \kappa_i/|a_i|$, which is the second form. An interval $(\alpha, 1-\beta)$ with $\alpha, \beta \geq 0$ is non-empty if and only if $\alpha + \beta < 1$, that is, $\kappa_i(1/c_{\mathrm{FR}} + 1/c_{\mathrm{FA}}) < |a_i|$. □

This is a classifier with a reject option [Chow, 1970]: commit when the posterior is extreme, defer when it is not. The last line is an endogenous materiality rule: with $c_{\mathrm{FA}} = c_{\mathrm{FR}} = m$, an item is worth verifying only if $|a_i| > 2\kappa_i/m$, whatever its probability.

Remark 5.3 (Buyer and seller hold different thresholds)**.** Consider positive addbacks. By Theorem 4.1 a false acceptance costs the buyer m per dollar and costs the seller nothing; a false rejection costs the seller m per dollar and costs the buyer at most the risk of losing the transaction. So the buyer has $c^B_{\mathrm{FA}} > c^B_{\mathrm{FR}}$ and a threshold $\theta_B > \frac{1}{2}$, and the seller has $c^S_{\mathrm{FA}} < c^S_{\mathrm{FR}}$ and a threshold $\theta_S < \frac{1}{2}$. Every addback with $\theta_S \leq p_i < \theta_B$ is accepted by a rational seller and rejected by a rational buyer *even when both agree on p_i*. The contested zone is a property of the losses, not of bad faith, and its price is $m \sum_{i:\,\theta_S \leq p_i < \theta_B} a_i$.

Proposition 5.4 (Expected-value bridge versus decided bridge)**.** *Let $\bar{A} = \mathbb{E}[A] = E_0 + \sum_i a_i p_i$ be the expected-value bridge. For any posterior and any number c that does not depend on z,*

$$\mathbb{E}\big[(c-A)^2\big] \;=\; \mathrm{Var}(A) + (c-\bar{A})^2. \tag{12}$$

Hence $\bar{A}$ is the unique minimiser of mean squared error, it is unbiased ($\mathbb{E}[\bar{A} - A] = 0$), and any decided bridge $\widehat{A}$ has bias

$$\widehat{A} - \bar{A} \;=\; \sum_{i:\, d_i = 1} a_i(1 - p_i) \;-\; \sum_{i:\, d_i = 0} a_i p_i \tag{13}$$

and excess mean squared error $(\widehat{A} - \bar{A})^2$.

Proof. Write $c - A = (c - \bar{A}) - (A - \bar{A})$, square, and take expectations; the cross term vanishes because $\mathbb{E}[A - \bar{A}] = 0$. This gives (12), which is minimised uniquely at $c = \bar{A}$. For (13), $\widehat{A} - \bar{A} = \sum_i a_i(d_i - p_i)$, and $d_i - p_i$ equals $1 - p_i$ when $d_i = 1$ and $-p_i$ when $d_i = 0$. □

The two bridges answer different questions. The decided bridge is the right object when each line must be a statement: a QoE report has to say whether an adjustment stands, and under the loss (9) the threshold rule is the best such statement. The expected-value bridge is the right object when a single number is to be priced and errors can net: it counts an addback with $p_i = 0.6$ at sixty cents on the dollar. Accepting every addback that is "more likely than not" and pricing on the total uses the first to do the job of the second; (13) is the resulting bias, of either sign.

6 Confidence Bands

A point estimate without a band invites false precision. For independent adjustments a distribution-free band follows from the inequality of Hoeffding [1963] for sums of independent bounded random variables: if $X_1, \dots, X_n$ are independent with $l_i \le X_i \le u_i$ and $S = \sum_i X_i$, then $\mathbb{P}(S - \mathbb{E}[S] \ge t) \le \exp(-2t^2 / \sum_i (u_i - l_i)^2)$ for every $t > 0$.

Theorem 6.1 (Hoeffding band for adjusted EBITDA)**.** *If the adjustments are independent, then for every $t > 0$,*

$$\mathbb{P}(|A - \mathbb{E}[A]| \ge t) \;\le\; 2\exp\!\left(-\frac{2t^2}{\sum_{i=1}^{n} a_i^2}\right). \tag{14}$$

Consequently, for $\delta \in (0, 1)$ and

$$t_\delta \;=\; \sqrt{\frac{\left(\sum_{i=1}^{n} a_i^2\right) \ln(2/\delta)}{2}}, \tag{15}$$

$\mathbb{P}(|A - \mathbb{E}[A]| < t_\delta) \ge 1 - \delta$ and, for a fixed multiple m, $\mathbb{P}(|V - m\mathbb{E}[A]| < m\, t_\delta) \ge 1 - \delta$.

Proof. The variables $X_i = a_i z_i$ are independent and each lies in an interval of length $|a_i|$ (namely $[0, a_i]$ or $[a_i, 0]$), so Hoeffding's inequality gives $\mathbb{P}(A - \mathbb{E}[A] \ge t) \le \exp(-2t^2 / \sum_i a_i^2)$. Applying it to $-X_i$ bounds the lower tail by the same quantity, and the union bound gives (14). Setting the right side equal to δ and solving for t gives (15). The statement for V follows from $V - m\mathbb{E}[A] = m(A - \mathbb{E}[A])$. □

Independence cannot be dropped from Theorem 6.1. Without it, Chebyshev's inequality $\mathbb{P}(|A - \mathbb{E}[A]| \ge t) \le \mathrm{Var}(A)/t^2$ with Proposition 3.2 still gives the half-width $\delta^{-1/2} \sum_i |a_i| \sqrt{p_i(1 - p_i)}$ at level δ, at a worse rate in δ; for uncorrelated items it gives $\delta^{-1/2}\, \mathrm{sd}(A) \le \frac{1}{2}\delta^{-1/2} (\sum_i a_i^2)^{1/2}$.

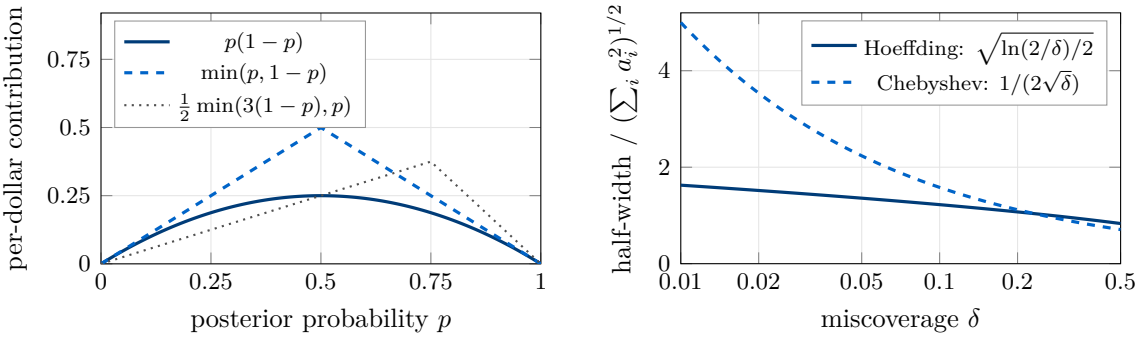


Figure 2: Left: what one dollar of an adjustment contributes as a function of its posterior probability: the variance factor $p(1-p)$ (solid), the minimal expected misclassification loss $\min(p, 1-p)$ under symmetric unit costs (dashed), and the same loss for a buyer with $c_{\mathrm{FA}} = 3c_{\mathrm{FR}}$, scaled by one half, which peaks at the buyer's threshold 0.75 (dotted). Right: half-width multipliers for independent adjustments, from (15) and from the worst-case Chebyshev form. Hoeffding is the smaller for δ below about 0.23.

Remark 6.2 (The bound is loose, and can be vacuous)**.** Hoeffding's inequality uses only the ranges $|a_i|$; it ignores p_i and so cannot see that an addback with $p_i = 0.97$ contributes almost no variance. It also competes with a trivial fact: A always lies in a feasible interval of length $\sum_i |a_i|$. For n adjustments of equal absolute size a, the band width $2t_\delta = a\sqrt{2n\ln(2/\delta)}$ is shorter than na if and only if $n > 2\ln(2/\delta)$; at $\delta = 0.05$ this requires $n \geq 8$. A short bridge has at most 2^n outcomes and should simply be enumerated, as in Section 10. The Hoeffding band earns its place for long bridges of comparable items, and as a guarantee that needs no probability inputs (Figure 2, right).

7 The Materiality Floor

Screening drops small candidates under a floor. Materiality is an established concept in auditing [International Auditing and Assurance Standards Board, 2009], where misstatements are considered individually *and in aggregate*. The following proposition makes the aggregate part exact for a floor of the form used in screening: one per cent of revenue, never less than 1,000.

Proposition 7.1 (What a floor does and does not bound)**.** *Let $R \geq 0$ be revenue and $\tau = \max(0.01\,R,\ 1000)$. Suppose $N \geq 1$ candidates are excluded because each has $|a_i| < \tau$. Then:*

(i) for every z, the omitted amount satisfies $\left|\sum_{i=1}^N a_i z_i\right| \leq \sum_{i=1}^N |a_i| < N\tau$;

(ii) the bound $N\tau$ is the best possible: for every $\varepsilon > 0$ there are admissible amounts with $\sum_{i=1}^N a_i > N\tau - \varepsilon$, and no admissible amounts attain $N\tau$;

(iii) for every level $M > 0$ and every $N > M/\tau$, there are admissible amounts, all valid, whose omitted total exceeds M.

Proof. (i) The triangle inequality and $z_i \in \{0,1\}$ give the first inequality; each of the $N \geq 1$ terms is strictly below τ, which gives the second. (ii) Take $a_i = \tau - \varepsilon'/(2N)$ for all i, with $\varepsilon' = \min(\varepsilon, N\tau)$; then $0 < a_i < \tau$ and $\sum_i a_i = N\tau - \varepsilon'/2 > N\tau - \varepsilon$. Attainment is excluded by (i). (iii) Since $N\tau > M$, apply (ii) with $\varepsilon = N\tau - M$ and set $z_i = 1$ for all i. □

A floor is a statement about each item and is silent about their number. From τ alone the omitted aggregate is unbounded, and by Theorem 4.1 every omitted valid dollar is worth m dollars of value. The count N restores a bound, $N\tau$, and the total absolute amount $T = \sum_i |a_i| \leq N\tau$ restores a better one. This is why the count and the total of excluded candidates must be reported alongside any floor: a reader who sees them can check in one line whether the exclusions are jointly immaterial, and a reader who does not see them cannot.

8 No Double Counting

Every result so far assumes that the amounts may be added. Two addbacks can claim the same invoice: a legal fee that appears under "litigation" and again under "non-recurring professional fees". To state the condition exactly, model the ledger.

Definition 8.1 (Row model)**.** Let $\mathcal{R}$ be a finite set of source rows with amounts $w : \mathcal{R} \to \mathbb{R}$, and write $w(S) = \sum_{r \in S} w(r)$ for $S \subseteq \mathcal{R}$. Adjustment i is supported on a row set $S_i \subseteq \mathcal{R}$ and has amount $a_i = w(S_i)$. The *multiplicity* of a row is $c(r) = |\{i : r \in S_i\}|$, and $U = \bigcup_i S_i$.

Proposition 8.2 (Additivity requires an overlap check)**.** *In the row model,*

$$\sum_{i=1}^{n} a_i \;-\; w(U) \;=\; \sum_{r\,:\,c(r)\geq 2} \big(c(r)-1\big)\, w(r). \tag{16}$$

Hence $\sum_i a_i = w(U)$ if and only if the right side of (16) is zero. If the rows of multiplicity at least two all have amounts of the same weak sign (all $w(r) \geq 0$, or all $w(r) \leq 0$), this holds if and only if the adjustments are pairwise disjoint on rows of non-zero amount, that is, $w(r) = 0$ for every $r \in S_i \cap S_j$, $i \neq j$. Without the sign condition, pairwise disjointness on non-zero rows is sufficient but not necessary. In general,

$$w(U) \;=\; \sum_{\emptyset \neq J \subseteq \{1,\ldots,n\}} (-1)^{|J|+1}\, w\Big(\bigcap_{j \in J} S_j\Big). \tag{17}$$

Proof. Exchanging the order of summation, $\sum_i a_i = \sum_i \sum_{r \in S_i} w(r) = \sum_{r \in U} c(r) w(r)$, while $w(U) = \sum_{r \in U} w(r)$. Subtracting gives $\sum_{r \in U} (c(r) - 1) w(r)$, and the rows with $c(r) = 1$ contribute nothing. This is (16). Under the sign condition every term on the right has the same weak sign and a strictly positive coefficient $c(r) - 1$, so the sum vanishes if and only if each $w(r)$ with $c(r) \geq 2$ vanishes; a row has $c(r) \geq 2$ exactly when it lies in some $S_i \cap S_j$ with $i \neq j$. Sufficiency without the sign condition is immediate. For non-necessity, let rows r_1, r_2 have $w(r_1) = 100$, $w(r_2) = -100$, and let $S_1 = S_2 = \{r_1, r_2\}$: then $\sum_i a_i = 0 = w(U)$ although both rows are claimed twice. For (17), a row with multiplicity $c \geq 1$ lies in $\bigcap_{j \in J} S_j$ for exactly $\binom{c}{k}$ sets J of size k, so its coefficient on the right is $\sum_{k=1}^{c} (-1)^{k+1} \binom{c}{k} = 1 - (1-1)^c = 1$; rows outside U have coefficient zero. □

For a bridge of cost addbacks the overlapping rows are expenses of one sign, so any overlap overstates adjusted EBITDA, and value by m times as much. The counterexample shows that checking totals cannot substitute for checking rows: a bridge can double count and still foot. The check requires knowing which source rows produced each adjustment, a lineage question of the kind studied in the database literature [Cui et al., 2000, Buneman et al., 2001].

9 Where to Verify

Review time is finite. Suppose verifying item i reveals z_i. I value a verification by the variance of A it removes, because by Proposition 5.4 that variance is, up to the factor m^2, the mean squared pricing error of the expected-value bridge.

Theorem 9.1 (Optimal verification under a count budget)**.** *Suppose the adjustments are independent, and let $S \subseteq \{1, \dots, n\}$ be the set of items verified. Then the residual variance is deterministic,*

$$\operatorname{Var}\left(A \mid (z_i)_{i \in S}\right) \;=\; \sum_{i \notin S} a_i^2 p_i (1 - p_i), \tag{18}$$

so verifying item i removes exactly $v_i = a_i^2 p_i (1 - p_i)$. Among all S with $|S| \le k$, the residual variance is minimised by any set consisting of k items with the largest values of v_i (or all items, if $k \ge n$). Because (18) does not depend on the revealed values, no adaptive policy that chooses items one at a time in light of earlier outcomes does better.

Proof. Conditional on $(z_i)_{i \in S}$, the terms $a_i z_i$ with $i \in S$ are constants, and by independence the remaining z_i are still independent with the same marginals p_i. Proposition 3.1 applied to the remaining items gives (18). Minimising $\sum_{i \notin S} v_i$ is maximising $\sum_{i \in S} v_i$ over sets of size at most k. Since $v_i \ge 0$, a maximiser may be taken of size $\min(k, n)$, and if S contains an item i while omitting an item j with $v_j > v_i$, exchanging them strictly increases the sum; so a maximiser consists of k largest values. An adaptive policy that verifies at most k items ends, on every path, with some verified set S of size at most k and residual variance $\sum_{i \notin S} v_i$, which is at least the minimum just found. □

Remark 9.2 (Large and genuinely uncertain)**.** The score $v_i = a_i^2 p_i (1 - p_i)$ is quadratic in size and, as Figure 2 (left) shows, maximal at $p_i = \frac{1}{2}$, where it equals $a_i^2/4$. It vanishes for items that are nearly certain either way. A large, well-documented item is a poor use of an hour, and so is a doubtful, trivial one. The decision-theoretic score of Proposition 5.2, $|a_i| \min(c_{\mathrm{FA}}(1 - p_i), c_{\mathrm{FR}}\, p_i)$, is linear in size and peaks at θ. The two rankings can differ (Section 10); which applies depends on whether the reviewer is pricing a total or deciding lines.

When items differ in the effort they take, let $\kappa_i > 0$ be the cost of verifying item i and $B > 0$ the budget. Choosing S to maximise $\sum_{i \in S} v_i$ subject to $\sum_{i \in S} \kappa_i \le B$ is the zero-one knapsack problem, whose linear relaxation is solved by a greedy rule due to Dantzig [1957].

Theorem 9.3 (Greedy optimality for the fractional relaxation)**.** *Let $v_i \geq 0$, $\kappa_i > 0$ and $B > 0$, and order the items so that $v_1/\kappa_1 \geq v_2/\kappa_2 \geq \cdots \geq v_n/\kappa_n$. Consider*

$$\max \sum_{i=1}^{n} v_i x_i \quad \textit{subject to} \quad \sum_{i=1}^{n} \kappa_i x_i \leq B, \quad 0 \leq x_i \leq 1. \tag{19}$$

If $\sum_i \kappa_i \leq B$, then $x^* = (1, \ldots, 1)$ is optimal. Otherwise let s be the first index with $\sum_{i \leq s} \kappa_i > B$; the greedy solution $x_i^* = 1$ for $i < s$, $x_s^* = (B - \sum_{i<s} \kappa_i)/\kappa_s$, $x_i^* = 0$ for $i > s$ is optimal. Moreover the integral set $\{1, \ldots, s-1\}$ is feasible for the zero-one problem and its value is within v_s of the zero-one optimum.

Proof. The first case is clear since $v_i \geq 0$. In the second, x^* is feasible with $\sum_i \kappa_i x_i^* = B$ and $x_s^* \in [0, 1)$. Let $\vartheta = v_s/\kappa_s \geq 0$ and let x be any feasible point. Then

$$\sum_i v_i(x_i - x_i^*) \;=\; \sum_i (v_i - \vartheta\kappa_i)(x_i - x_i^*) \;+\; \vartheta \sum_i \kappa_i (x_i - x_i^*).$$

For $i < s$, $v_i - \vartheta\kappa_i \geq 0$ by the ordering and $x_i - x_i^* = x_i - 1 \leq 0$. For $i > s$, $v_i - \vartheta\kappa_i \leq 0$ and $x_i - x_i^* = x_i \geq 0$. For $i = s$ the first factor is zero. So the first sum is non-positive. The second is non-positive because $\sum_i \kappa_i x_i \leq B = \sum_i \kappa_i x_i^*$. Hence $\sum_i v_i x_i \leq \sum_i v_i x_i^*$. For the last claim, the zero-one optimum is at most the optimum of (19), which equals $\sum_{i<s} v_i + v_s x_s^* \leq \sum_{i<s} v_i + v_s$. □

So the practical rule is to rank by variance removed per unit of review cost, $a_i^2 p_i(1-p_i)/\kappa_i$: exactly optimal if effort is divisible, and within one item's worth otherwise.

10 An Illustrative Worked Example

The following bridge is invented for illustration and describes no real company. All amounts are in thousands of US dollars. Reported EBITDA is $E_0 = 4{,}000$, revenue is 20,000, and the multiple is $m = 6$. Six addbacks pass the floor $\tau = \max(0.01 \times 20{,}000,\ 1) = 200$. Table 1 lists them with assumed posterior probabilities.

Table 1: An illustrative bridge (USD thousands): amount a_i, posterior probability p_i, variance contribution $v_i = a_i^2 p_i(1-p_i)$, $s_i = \sqrt{v_i}$, symmetric expected misclassification $a_i \min(p_i, 1-p_i)$ (column $a_i$min), and rank by v_i.

i	Adjustment	a_i	p_i	$a_i p_i$	v_i	s_i	$a_i$min	Rank
1	Owner compensation above market	600	0.90	540	32,400	180.00	60	3
2	Litigation settlement, one-time	400	0.80	320	25,600	160.00	80	4
3	ERP implementation consulting	300	0.60	180	21,600	146.97	120	5
4	Related-party rent normalisation	250	0.50	125	15,625	125.00	125	6
5	Discontinued product line loss	500	0.40	200	60,000	244.95	200	2
6	Pro forma price increase	800	0.30	240	134,400	366.61	240	1
	Total	2,850		1,605	289,625	1,223.52	825	

Moments. By Proposition 3.1, $\mathbb{E}[A] = 4{,}000 + 1{,}605 = 5{,}605$, against a seller's headline of $4{,}000 + 2{,}850 = 6{,}850$. Under independence $\mathrm{Var}(A) = 289{,}625$ and $\mathrm{sd}(A) = \sqrt{289{,}625} \approx 538.17$. The dependence-free bound of Proposition 3.2 is $\mathrm{sd}(A) \le 1{,}223.52$. The sharp same-sign bound of Proposition 3.3 evaluates to $\mathrm{Var}(A) \le 926{,}225$, that is, $\mathrm{sd}(A) \le 962.41$: under the comonotone coupling A takes the nested values 4,000, 4,600, 5,000, 5,300, 5,550, 6,050, 6,850 with probabilities 0.1, 0.1, 0.2, 0.1, 0.1, 0.1, 0.3. A common narrative can therefore raise the standard deviation by a factor of about 1.79 without changing a single p_i. In value terms, $\mathbb{E}[V] = 6 \times 5{,}605 = 33{,}630$ with independent standard deviation $6 \times 538.17 \approx 3{,}229$, against a headline of $6 \times 6{,}850 = 41{,}100$.

Decided bridges. Under symmetric loss the threshold is $\frac{1}{2}$ and items 1 to 4 are accepted (item 4 is a tie, accepted by convention): $\widehat{A} = 4{,}000 + 1{,}550 = 5{,}550$ and $\widehat{V} = 33{,}300$. The bias (13) is $5{,}550 - 5{,}605 = -55$, and the mean squared error is $289{,}625 + 55^2 = 292{,}650$ against 289,625 for the expected-value bridge. A buyer with $c_{\mathrm{FA}} = 3c_{\mathrm{FR}}$ has $\theta_B = 0.75$ and accepts only items 1 and 2: $\widehat{A}_B = 5{,}000$, $\widehat{V}_B = 30{,}000$. A seller with $c_{\mathrm{FR}} = 3c_{\mathrm{FA}}$ has $\theta_S = 0.25$ and accepts all six: $\widehat{A}_S = 6{,}850$, $\widehat{V}_S = 41{,}100$. The contested zone of Remark 5.3 is items 3 to 6, worth 1,850 of EBITDA and 11,100 of value, with no disagreement about any probability.

Bands. Here $\sum_i a_i^2 = 1{,}562{,}500 = 1{,}250^2$, so (15) gives $t_{0.05} = 1{,}250\sqrt{\ln(40)/2} \approx 1{,}697.6$ and the band $[3{,}907.4,\ 7{,}302.6]$ for A, or $\pm 10{,}186$ on V. The feasible interval is $[4{,}000,\ 6{,}850]$, so with six items the 95 per cent Hoeffding band is vacuous ($6 < 2\ln 40 \approx 7.38$). Enumerating the $2^6 = 64$ outcomes under independence gives the exact statement $\mathbb{P}(4{,}600 \le A \le 6{,}600) = 0.95784$: a 95 per cent interval of half-width 1,000, not 1,698. The seller's headline, in which every addback is valid, has probability $0.9 \times 0.8 \times 0.6 \times 0.5 \times 0.4 \times 0.3 = 0.02592$.

Verification. Ranking by v_i gives the order $6, 5, 1, 2, 3, 4$. With a budget of two items, Theorem 9.1 selects items 6 and 5, removing 194,400 of 289,625 (about 67.1 per cent) and leaving a standard deviation of $\sqrt{95{,}225} \approx 308.59$. Item 4, the most uncertain line ($p = 0.5$), ranks last because it is small; item 1, the best supported ($p = 0.9$), ranks third because it is large. The decision-loss score $a_i \min(p_i, 1 - p_i)$ gives a different order, $6, 5, 4, 3, 2, 1$. With unequal review costs of $\kappa = (2, 4, 3, 1, 8, 16)$ hours and a budget of $B = 20$ hours, the ratios v_i/κ_i are 16,200, 6,400, 7,200, 15,625, 7,500, 8,400, so the greedy order is $1, 4, 6, 5, 3, 2$. Items 1, 4 and 6 use 19 hours and remove 182,425; the fractional optimum of Theorem 9.3 adds one eighth of item 5 for 189,925. Exhaustive search confirms that $\{1, 4, 6\}$ is also the zero-one optimum here, leaving $\mathrm{sd}(A) = \sqrt{107{,}200} \approx 327.41$.

Floor and overlap. If 14 further candidates were excluded under $\tau = 200$, Proposition 7.1 bounds their aggregate only by 2,800, worth up to 16,800 of value. If items 2 and 3 both include the same invoice of 120, (16) shows the bridge overstated by 120 and the value by 720.

11 Implications for Practice

The theory separates what can be computed from what must be judged. E_0, the amounts a_i and the row sets S_i are deterministic functions of the ledger; the p_i are not. Finsider's design follows the same line: it computes a deterministic core from ledger data, and any figure extracted by an AI layer is kept separate, labelled as needing verification, and does not feed headline metrics until verified. In the language of this paper, such a figure has p_i strictly inside $(0, 1)$, and keeping it out of the headline is the conservative end of Theorem 5.1. Proposition 8.2 presupposes row-level lineage: Finsider keeps immutable original source files and traces reported figures to source rows. Proposition 7.1 is the reason its screening floor (one per cent of mapped revenue, never below USD 1,000) is reported with the count and total of the candidates excluded under it. Proposition 5.2 describes a release discipline: a company's results are held behind a release gate until verification passes, with human reviewers in the loop.

12 Falsifiable Predictions

The following are predictions about what a future study with access to bridges, evidence files and post-close outcomes would find. None is a reported result.

Prediction 1 (Evidence grades are calibratable)**.** Addbacks graded by a fixed, pre-registered evidence rubric into probability bins will survive buy-side diligence and post-close true-up at rates that increase monotonically across bins. *Refuted if* survival rates are flat or non-monotone across bins in a sample large enough to detect a ten-point difference.

Prediction 2 (Addback failures cluster within a deal)**.** Within a bridge, the outcomes of addbacks that share a source (the same management representation or preparer) are positively correlated, so the realised dispersion of A exceeds the independent figure $\sum_i a_i^2 p_i(1-p_i)$. *Refuted if* the estimated within-deal pairwise correlation of outcomes is indistinguishable from zero.

Prediction 3 (Disputes concentrate in the contested zone)**.** Addbacks that become the subject of price negotiation are disproportionately those with intermediate evidence grades, as Remark 5.3 implies. *Refuted if*, controlling for amount, the probability that an addback is disputed does not peak at intermediate grades.

Prediction 4 (Variance ranking beats size ranking)**.** For a fixed review budget, ordering review by $a_i^2 p_i(1-p_i)$ per unit cost identifies more dollars of subsequently reversed addbacks than ordering by $|a_i|$ alone. *Refuted if* a size-only ordering finds as many reversed dollars per review hour in a controlled comparison.

13 Limitations

The probabilities must come from somewhere. Everything here is conditional on p_i, and a confident wrong p_i is worse than an admitted unknown. Calibration is an empirical property that can fail: even high-capacity statistical models are miscalibrated unless corrected

[Guo et al., 2017]. Ground truth for addback validity is scarce and late, and estimating reviewer error rates without it is itself a statistical problem [Dawid and Skene, 1979]. Until a calibration study exists, the p_i are structured judgement and sensitivity to them should be reported.

Evidence is not exogenous. The seller chooses what to show. A posterior that conditions on the evidence presented, and not on the fact of its selection, will be too generous. This is the lemons problem [Akerlof, 1970] inside the data room, and the model treats it only through whatever discount the analyst builds into p_i.

The multiple is negotiated and uncertain. I held m fixed. In practice m is bargained, uncertain, and plausibly correlated with perceived earnings quality, so that a weak bridge lowers both A and m. Then $V = mA$ is a product of dependent random quantities and Theorem 4.1 gives only the partial effect of A at fixed m.

Independence. The variance formula, Theorem 6.1 and Theorem 9.1 assume it. Remark 3.4 argues that it is the optimistic assumption. Under dependence the variance reduction is no longer a sum of per-item scores, and selecting what to observe becomes a combinatorial problem [Krause and Guestrin, 2005]; the qualitative advice is to verify a shared premise first. Propositions 3.2 and 3.3 are the fallback, and they are wide.

Binary validity and linear loss. If $z_i \in [0, 1]$ is a valid fraction with mean p_i, the mean formula and Theorem 6.1 hold unchanged (the range of $a_i z_i$ is still $|a_i|$), and $a_i^2 p_i(1 - p_i)$ becomes an upper bound on the variance of the term, since $z_i^2 \le z_i$; the acceptance theory does not carry over as stated. A party with a fixed cost of dispute, or with risk aversion, has a threshold that depends on a_i.

14 Conclusion

Once each addback is given a probability, the rest is forced. The expected bridge is $E_0 + \sum_i a_i p_i$. Its variance is $\sum_i a_i^2 p_i(1 - p_i)$ under independence and can be several times larger under a common narrative. An acceptance error of a dollars moves price by ma. The optimal threshold is the cost ratio $c_{\mathrm{FA}}/(c_{\mathrm{FA}} + c_{\mathrm{FR}})$, so a buyer and a seller rationally disagree over a predictable band of items. A floor bounds items, not totals. A sum of addbacks means what it says only if no non-zero row is claimed twice. An hour of review is worth most on the item that is both large and genuinely uncertain. None of this replaces judgement about a particular addback; it says where that judgement carries the most weight.

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