The Working-Capital Peg as an Estimation Problem: Seasonality, Window Choice, and Who Pays for the Error

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October 2026

Abstract

In most private acquisitions the price moves dollar for dollar with the difference between the net working capital delivered at closing and a number agreed in advance, the peg. I treat the peg as what it is: an estimator of a quantity nobody observes, the normal level of working capital at the closing date. Because the adjustment is linear with slope one, every dollar of estimation error is a dollar transferred between buyer and seller. Under an additive seasonal model I derive the exact bias and variance of the trailing k-month average, show that the trailing twelve months is unbiased for the annual mean under every seasonal pattern but is aimed at the wrong target, and show that the expected transfer it creates equals the seasonal component of the closing month. With a linear trend the window length becomes a bias-variance choice with a closed first-order condition, and I give the exact thresholds at which a 3, 6 or 12 month base is best. I bound how far one distorted month moves a mean, a trimmed mean and a median; prove that the price formula is neutral to timing between cash and working capital if and only if the contractual definitions cover the affected balance-sheet items exactly once; and derive the expected adjustment under a symmetric collar in closed form. The paper is theory and method: it reports no empirical results, does not model bargaining, and holds only as far as the stated model does. I close with five falsifiable predictions.

1 Introduction

A buyer who agrees an enterprise value is paying for a going concern, and a going concern needs a normal amount of working capital to operate. The purchase agreement therefore fixes a reference level, the *peg* (also called the target), and adjusts the price by the difference between the net working capital delivered at closing and that reference. If the seller delivers more than the peg, the buyer pays for the excess. If the seller delivers less, the price falls.

The arithmetic is not controversial. The number is. The usual practice is to average monthly balances over a trailing window of 3, 6 or 12 months, sometimes after removing months judged abnormal, and to negotiate from there. The practice is rarely described as what it is. The peg is an *estimate*. Its target is the level of working capital the business

would normally hold on the closing date, a quantity that is never observed, because the closing balance that eventually arrives is one noisy draw and arrives after the peg is fixed. An estimator has a bias and a variance, both of which can be computed, and because the price adjustment is linear with slope one the error of the estimator is paid in cash by one side to the other.

This paper works that observation through: the exact bias and variance of trailing averages (Section 3), the target they miss (Section 4), window choice under a trend (Section 5), robustness (Section 6), neutrality to timing (Section 7), collars (Section 8), an operating identity (Section 9) and a worked example (Section 10). There is no data here and no claim about how pegs are in fact set. A negotiated peg reflects bargaining power and information as well as estimation, and I model only the estimation; Section 13 says where the model is likely to fail.

Related work. Working-capital accruals are estimates of future cash flows, and the accounting literature has long studied the error in them: Sloan [1996] on the lower persistence of the accrual component of earnings, Dechow and Dichev [2002] on accrual quality as the mapping of working-capital accruals into realised cash flows, Richardson et al. [2005] on accrual reliability, and the review of Dechow et al. [2010]. Jones [1991] decomposed accruals into a normal component predicted from fundamentals and a discretionary remainder, and Beneish [1999] used a days-sales-in-receivables index among the signals of manipulation. The peg problem is the same decomposition inside a transaction: the parties must agree a normal level before the realised level is known, and the seller knows the business better, the setting of Akerlof [1970]. A first proposal for the peg also acts as an anchor in the sense of Tversky and Kahneman [1974]. The statistical tools are classical: seasonal decomposition [Cleveland et al., 1990] and robust estimation of location [Huber, 1964, Hampel, 1974].

2 Setup: the Price Mechanism and the Model

Time is measured in months. Let W_t denote net working capital at the end of month t, computed under the definition in the purchase agreement. The last balance sheet available when the peg is fixed is month T, and the deal closes at the end of month $T + d$ with $d \geq 1$. Write $m(t) \in \{1, \ldots, 12\}$ for the calendar month of t and $c = m(T + d)$ for the calendar month of closing.

Definition 2.1 (Price mechanism)**.** In a cash-free, debt-free transaction with enterprise value EV and peg P, the equity price is

$$\text{Eq} \;=\; \text{EV} \;-\; \text{Debt}_{T+d} \;+\; \text{Cash}_{T+d} \;+\; (W_{T+d} - P). \tag{1}$$

The peg P is a function of information available at T only.

Definition 2.2 (Additive seasonal model)**.** There are a level $\mu \in \mathbb{R}$, seasonal components $s_1, \ldots, s_{12} \in \mathbb{R}$ with $\sum_{j=1}^{12} s_j = 0$, and mean-zero random variables ε_t with common variance $\sigma^2 < \infty$ such that

$$W_t \;=\; \mu + s_{m(t)} + \varepsilon_t. \tag{2}$$

Under assumption (IID) the ε_t are independent and identically distributed. Under assumption (WS) they are only weakly stationary with autocovariance $\gamma(h) = \text{Cov}(\varepsilon_t, \varepsilon_{t+h})$, $\gamma(0) = \sigma^2$.

Definition 2.3 (Target and estimation error)**.** The *normal level at close* is $\tau_c = \mathbb{E}[W_{T+d}]$, which under (2) equals $\mu + s_c$. The *estimation error* of a peg is $e = P - \tau_c$.

The decomposition that organises the paper is immediate from (1):

$$W_{T+d} - P \;=\; \underbrace{(W_{T+d} - \tau_c)}_{\text{delivered against normal}} \;-\; \underbrace{(P - \tau_c)}_{\text{estimation error } e}. \tag{3}$$

The first term is what the clause exists to price: the seller delivered more or less than normal. The second is the error in the reference, and it enters the price with coefficient -1. A peg that is too high by one dollar costs the seller one dollar; a peg that is too low by one dollar costs the buyer one dollar.

3 The Trailing Average: Exact Bias and Variance

The trailing k-month average ending at T is $\overline{W}_k(T) = \frac{1}{k}\sum_{i=0}^{k-1} W_{T-i}$. Define the *window seasonal mean*

$$B_k(T) \;=\; \frac{1}{k}\sum_{i=0}^{k-1} s_{m(T-i)}. \tag{4}$$

Theorem 3.1 (Trailing average)**.** *Under the model (2), for every $k \geq 1$, $\mathbb{E}[\overline{W}_k(T)] = \mu + B_k(T)$. In particular $\mathbb{E}[\overline{W}_{12}(T)] = \mu$ for every seasonal pattern and every T. Under* (IID)*, $\mathrm{Var}(\overline{W}_k(T)) = \sigma^2/k$, so the trailing twelve months has variance $\sigma^2/12$.*

Proof. By linearity, $\mathbb{E}[\overline{W}_k(T)] = \mu + \frac{1}{k}\sum_i s_{m(T-i)} + \frac{1}{k}\sum_i \mathbb{E}[\varepsilon_{T-i}] = \mu + B_k(T)$. Twelve consecutive months contain each calendar month exactly once, so $B_{12}(T) = \frac{1}{12}\sum_{j=1}^{12} s_j = 0$. Under (IID) the variance of a mean of k independent terms of variance σ^2 is σ^2/k. □

The unbiasedness for μ needs no assumption on dependence. The variance does.

Lemma 3.2 (Variance under dependence)**.** *Under* (WS), $\mathrm{Var}(\overline{W}_k(T)) = \frac{1}{k^2}\sum_{h=-(k-1)}^{k-1}(k - |h|)\,\gamma(h)$. *In particular, if $\gamma(h) \geq 0$ for all h and $\gamma(h) > 0$ for some $1 \leq h < k$, the variance strictly exceeds σ^2/k.*

Proof. $\mathrm{Var}(\sum_{i=0}^{k-1}\varepsilon_{T-i}) = \sum_{i,j}\gamma(i-j)$, and the lag $h = i - j$ occurs for exactly $k - |h|$ pairs. The term $h = 0$ contributes $k\sigma^2$ and the others are non-negative under the stated condition, with at least one positive. □

Balances carry over from month to month, so positive autocorrelation is the natural case: a twelve-month average of persistent noise is worth fewer than twelve independent observations.

Proposition 3.3 (Bias of short windows)**.** *Let $1 \leq k \leq 11$.*

(a) $\sum_{T=1}^{12} B_k(T) = 0$*, where the sum runs over twelve consecutive end months. Hence either $B_k(T) = 0$ for every end month, or the k-month average is biased upward for some end months and downward for others.*

(b) $k\,B_k(T) = -(12-k)\,B_{12-k}(T-k)$.

(c) *If $|s_j| \le A$ for all j, then $|B_k(T)| \le A \min\{1, (12-k)/k\}$, and for each k the bound is attained by some seasonal pattern.*

Proof. (a) Each calendar month lies in exactly k of the twelve windows, so $\sum_T k\, B_k(T) = k \sum_j s_j = 0$. (b) The k months ending at T and the $12-k$ months ending at $T-k$ together make twelve consecutive months, whose seasonal components sum to zero. (c) The bound $|B_k(T)| \le A$ is the triangle inequality, and by (b), $|B_k(T)| = \frac{12-k}{k}|B_{12-k}(T-k)| \le \frac{12-k}{k}A$. For attainment when $k \le 6$, set $s_j = A$ on the k window months and $s_j = -kA/(12-k)$ on the other $12-k$ months; then $|s_j| \le A$ and $B_k(T) = A$. When $k > 6$, set $s_j = -A$ on the $12-k$ months outside the window and $s_j = (12-k)A/k$ inside it; then $B_k(T) = (12-k)A/k$. □

For a 3 or 6 month base the worst case is the full seasonal amplitude: a short window can sit entirely inside a peak. For a smooth pattern the bias has a closed form.

Proposition 3.4 (Sinusoidal seasonality)**.** *Number the months so that $m(t) \equiv t \pmod{12}$. If $s_j = A\cos\big(2\pi(j-\phi)/12\big)$, then*

$$B_k(T) = A\, D_k \cos\Big(\frac{2\pi}{12}\Big(T - \frac{k-1}{2} - \phi\Big)\Big), \qquad D_k = \frac{\sin(\pi k/12)}{k\,\sin(\pi/12)}.$$

Hence $|B_k(T)| \le A\, D_k$, with $D_3 \approx 0.911$, $D_6 \approx 0.644$, $D_9 \approx 0.304$ and $D_{12} = 0$.

Proof. With $\theta = 2\pi/12$ and $a = \theta(T-\phi)$, the identity $\sum_{i=0}^{k-1}\cos(a - i\theta) = \frac{\sin(k\theta/2)}{\sin(\theta/2)}\cos\big(a - (k-1)\theta/2\big)$ for a sum of cosines in arithmetic progression gives the result after division by k. The pattern sums to zero over twelve months because $D_{12} = 0$. □

A three-month average of a sinusoidal pattern thus retains up to 91 per cent of the seasonal amplitude as bias and a six-month average up to 64 per cent, depending only on where the centre of the window falls relative to the peak.

4 The Target Is Not the Annual Mean

Theorem 3.1 is the usual justification for the trailing twelve months: it averages out the seasons. That is correct and it is the problem. The adjustment compares one particular month, not the year, with the peg.

Theorem 4.1 (Expected transfer)**.** *Under the model (2), a peg $P = \overline{W}_k(T)$ produces an expected price adjustment*

$$\mathbb{E}\big[W_{T+d} - \overline{W}_k(T)\big] = s_c - B_k(T).$$

For $k = 12$ the expected adjustment is exactly s_c: a transfer to the seller when the close falls in a month with $s_c > 0$ and to the buyer when $s_c < 0$. Any peg with $\mathbb{E}[P] = \mu + s_c$ has expected adjustment zero. If the closing month is drawn uniformly from the twelve calendar months, the mean squared expected transfer under the twelve-month peg is the seasonal variance $\frac{1}{12}\sum_{j=1}^{12} s_j^2$.

Proof. $\mathbb{E}[W_{T+d}] = \mu + s_c$ and $\mathbb{E}[\overline{W}_k(T)] = \mu + B_k(T)$ by Theorem 3.1; subtract. For $k = 12$, $B_{12}(T) = 0$. The last claim is the average of s_c^2 over c. □

The expected transfer is fixed by the calendar. Under the twelve-month peg, a seller who can choose to close at a seasonal peak collects s_c in expectation, and a buyer who can push the close into a trough collects $|s_c|$. A short window does not repair this: it replaces s_c with $s_c - B_k(T)$, which vanishes only when the window happens to resemble the closing month.

4.1 Estimating the aligned target from limited history

The natural unbiased estimator of $\mu + s_c$ is the average of the same calendar month in earlier years. Suppose the history consists of H complete years, the $12H$ months ending at T. Let $\overline{W}$ be the grand mean of all $12H$ months and $\overline{W}^{(c)}$ the mean of the H observations of calendar month c.

Proposition 4.2 (Aligned peg from H years)**.** *Under* (2) *and* (IID)*:*

(a) $\overline{W}^{(c)}$ *is unbiased for* $\tau_c = \mu + s_c$ *with variance* σ^2/H*.*

(b) The seasonal estimate $\hat{s}_c = \overline{W}^{(c)} - \overline{W}$ *is unbiased for* s_c *with variance* $\frac{11}{12}\,\sigma^2/H$*, and is uncorrelated with* $\overline{W}$*, whose variance is* $\sigma^2/(12H)$*.*

(c) For the family $P_\lambda = \overline{W} + \lambda\,\hat{s}_c$*,* $\lambda \in [0,1]$*,*

$$\mathbb{E}\big[(P_\lambda - \tau_c)^2\big] = (1-\lambda)^2 s_c^2 + \frac{\sigma^2}{12H}\big(1 + 11\lambda^2\big),$$

minimised at $\lambda^* = s_c^2/\big(s_c^2 + \frac{11}{12}\sigma^2/H\big)$*.*

(d) The fully aligned peg ($\lambda = 1$) *has smaller mean squared error than the annual-average peg* ($\lambda = 0$) *if and only if* $s_c^2 > \frac{11}{12}\,\sigma^2/H$*.*

Proof. (a) $\overline{W}^{(c)}$ is a mean of H independent observations each with mean $\mu + s_c$ and variance σ^2. (b) $\mathbb{E}[\overline{W}] = \mu$ because the seasonal components sum to zero over each year, and $\mathrm{Var}(\overline{W}) = \sigma^2/(12H)$. Since $\overline{W}^{(c)}$ shares exactly H of the $12H$ terms of $\overline{W}$, $\mathrm{Cov}(\overline{W}^{(c)}, \overline{W}) = \frac{1}{H}\cdot\frac{1}{12H}\cdot H\sigma^2 = \sigma^2/(12H)$. Therefore $\mathrm{Var}(\hat{s}_c) = \sigma^2/H + \sigma^2/(12H) - 2\sigma^2/(12H) = \frac{11}{12}\sigma^2/H$ and $\mathrm{Cov}(\hat{s}_c, \overline{W}) = \sigma^2/(12H) - \sigma^2/(12H) = 0$. (c) The bias of P_λ is $\mu + \lambda s_c - \mu - s_c = -(1-\lambda)s_c$, and by (b) its variance is $\sigma^2/(12H) + \lambda^2\frac{11}{12}\sigma^2/H$. Setting the derivative in λ to zero gives $-(1-\lambda)s_c^2 + \lambda\frac{11}{12}\sigma^2/H = 0$. (d) Compare the values σ^2/H at $\lambda = 1$ and $s_c^2 + \sigma^2/(12H)$ at $\lambda = 0$. □

Three points follow. First, the level and seasonal terms do not add their variances naively: $\overline{W} + \hat{s}_c$ collapses to $\overline{W}^{(c)}$, whose variance σ^2/H is twelve times that of the grand mean. Alignment buys zero bias at the price of using one observation per year. Second, with a single year of history the aligned peg is last year's balance for the closing month, and it beats the annual average only when $|s_c| > 0.957\,\sigma$. Third, λ^* depends on the unknown s_c, so (c) describes an oracle, not a procedure; it shows that partial seasonal adjustment is the efficient compromise.

When the seasonal pattern drifts or a trend is present, same-month averages are no longer adequate. The standard tool is the STL procedure of Cleveland et al. [1990], which

decomposes a series into trend, seasonal and remainder components by repeated loess smoothing, lets the seasonal component evolve, and includes robustness iterations that reduce the influence of aberrant observations. STL needs several full cycles, and the monthly history supplied in a transaction is often two or three years. I claim no finite-sample property for STL on series that short; Proposition 4.2 is the exact statement available under the simple model.

5 Trend and the Bias-Variance Choice of Window

Businesses that are sold are often growing, and working capital grows with them (Section 9). Remove seasonality for the moment and let

$$W_t = \mu + g\,t + \varepsilon_t, \tag{5}$$

with g the monthly drift and (IID) noise. The level at time t is $L_t = \mu + gt$, and the target at close is L_{T+d}.

Theorem 5.1 (Lag and optimal window)**.** *Under* (5)*,* $\mathbb{E}[\overline{W}_k(T)] = L_T - g\,(k-1)/2$ *and* $\mathrm{Var}(\overline{W}_k(T)) = \sigma^2/k$*, so for a target* L_{T+d} *with* $d \geq 0$*,*

$$\mathrm{MSE}_d(k) = g^2\Big(\frac{k-1}{2} + d\Big)^2 + \frac{\sigma^2}{k}. \tag{6}$$

Let $g \neq 0$ *and* $r = \sigma^2/g^2$*. Regarded as a function of real* $k \geq 1$*,* MSE_d *is strictly convex. If* $r > d$ *it has a unique minimiser* $k^* > 1$*, the unique root in* $(1, \infty)$ *of the cubic*

$$k^2\,(k - 1 + 2d) = 2r; \tag{7}$$

if $r \leq d$ *the minimiser is* $k = 1$*. For* $d = 0$ *and* $r > 0$ *the minimiser satisfies* $(2r)^{1/3} < k^* < (2r)^{1/3} + 1$*, and* $k^* = (2r)^{1/3} + \frac{1}{3} + o(1)$ *as* $r \to \infty$*.*

Proof. The mean of t over $t = T-k+1, \ldots, T$ is $T - (k-1)/2$, which gives the expectation, and the variance is as in Theorem 3.1. The bias against L_{T+d} is $-g((k-1)/2+d)$, giving (6). Differentiating, $\mathrm{MSE}'_d(k) = g^2((k-1)/2+d) - \sigma^2/k^2$ and $\mathrm{MSE}''_d(k) = g^2/2 + 2\sigma^2/k^3 > 0$, so the function is strictly convex and its derivative is strictly increasing. Setting the derivative to zero and multiplying by $2k^2/g^2$ gives (7). At $k = 1$ the derivative is $g^2 d - \sigma^2$, negative exactly when $r > d$, and the derivative tends to $+\infty$; hence a unique interior root when $r > d$, and a minimum at the boundary $k = 1$ otherwise. For $d = 0$ the equation is $k^2(k-1) = 2r$. Since $k^{*3} > k^{*2}(k^* - 1) = 2r$ we get $k^* > (2r)^{1/3}$, and since $(k^* - 1)^3 < k^{*2}(k^* - 1) = 2r$ we get $k^* < (2r)^{1/3} + 1$. Finally $k^* = (2r)^{1/3}(1 - 1/k^*)^{-1/3} = (2r)^{1/3}(1 + \frac{1}{3k^*} + O(k^{*-2}))$, and $(2r)^{1/3}/k^* \to 1$, which gives the expansion. □

The optimal window grows slowly with the noise-to-drift ratio: doubling $\sigma/|g|$ lengthens it by a factor of about $2^{2/3} \approx 1.59$. Figure 1 plots (6) for three values of r. In practice the choice is among three bases, and the comparison is exact.

Corollary 5.2 (When the 3, 6 or 12 month base wins)**.** *Under* (5) *with* $g \neq 0$*,* $r = \sigma^2/g^2$ *and lead* $d \geq 0$*, among* $k \in \{3, 6, 12\}$ *the 3-month base has the smallest mean squared error when* $r < 31.5 + 18d$*, the 6-month base when* $31.5 + 18d < r < 288 + 72d$*, and the 12-month base when* $r > 288 + 72d$*. For* $d = 0$ *the thresholds in* $\sigma/|g|$ *are* $\sqrt{31.5} \approx 5.61$ *and* $\sqrt{288} \approx 16.97$*.*

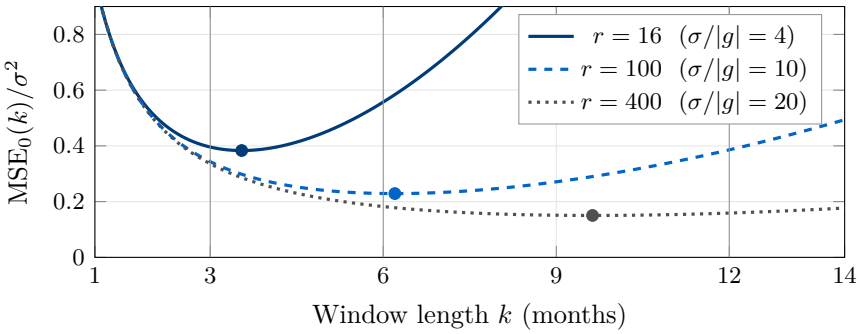


Figure 1: The mean squared error (6) of a trailing k-month peg under a linear trend with $d = 0$, in units of σ^2, for three values of $r = \sigma^2/g^2$. Dots mark the minimisers given by (7): $k^* \approx 3.55$, 6.20 and 9.63. Vertical lines mark the 3, 6 and 12 month bases; consistent with Corollary 5.2, each is best for one of the curves.

Proof. From (6), $\mathrm{MSE}_d(3) < \mathrm{MSE}_d(6)$ if and only if $\sigma^2/6 < g^2[(2.5+d)^2 - (1+d)^2] = g^2(5.25 + 3d)$, that is $r < 31.5 + 18d$. Likewise $\mathrm{MSE}_d(6) < \mathrm{MSE}_d(12)$ if and only if $\sigma^2/12 < g^2[(5.5+d)^2 - (2.5+d)^2] = g^2(24 + 6d)$, that is $r < 288 + 72d$, and $\mathrm{MSE}_d(3) < \mathrm{MSE}_d(12)$ if and only if $\sigma^2/4 < g^2[(5.5+d)^2 - (1+d)^2] = g^2(29.25 + 9d)$, that is $r < 117 + 36d$. Since $31.5 + 18d < 117 + 36d < 288 + 72d$ for all $d \geq 0$, the three regions are as stated. □

5.1 Trend and seasonality together

Now combine the two: $W_t = \mu + gt + s_{m(t)} + \varepsilon_t$, with target $\tau = \mu + g(T+d) + s_c$.

Proposition 5.3 (The combined case)**.** *Under the combined model with* (IID) *noise:*

(a) The trailing k-month peg has bias $B_k(T) - s_c - g\left(\frac{k-1}{2} + d\right)$ and variance σ^2/k. For $k = 12$ the bias is $-s_c - g\,(5.5 + d)$.

(b) Let $1 \leq d \leq 12$, let M_1 be the mean of the twelve months ending at T and M_2 the mean of the twelve months before those. The seasonal-difference peg

$$P^{\mathrm{sd}} = W_{T+d-12} + (M_1 - M_2)$$

is unbiased for τ and has variance $\frac{4}{3}\sigma^2$.

(c) P^{sd} has smaller mean squared error than the twelve-month peg if and only if $\left(s_c + g\,(5.5+d)\right)^2 > \frac{5}{4}\sigma^2$.

Proof. (a) Expectation is linear, so the trend lag of Theorem 5.1 and the seasonal mean of Theorem 3.1 add. (b) Month $T + d - 12$ has calendar month c, so $\mathbb{E}[W_{T+d-12}] = \mu + g(T + d - 12) + s_c$. Each of M_1, M_2 averages a full year, so the seasonal terms vanish and $\mathbb{E}[M_1 - M_2] = 12g$. The sum is τ. For the variance, W_{T+d-12} is one of the twelve terms of M_1 when $1 \leq d \leq 12$ and is independent of M_2, and M_1, M_2 are independent, so $\mathrm{Var}(P^{\mathrm{sd}}) = \sigma^2 + \frac{\sigma^2}{12} + \frac{\sigma^2}{12} + 2 \cdot \frac{\sigma^2}{12} = \frac{4}{3}\sigma^2$. (c) Compare $\frac{4}{3}\sigma^2$ with $(s_c + g(5.5+d))^2 + \frac{1}{12}\sigma^2$. □

Part (a) is the honest summary of the twelve-month convention. It removes the window seasonal mean by adopting the longest lag on offer: at $d = 0$ it trails the current level by $5.5\,g$, against $2.5\,g$ and $1\,g$ for the 6 and 3 month bases. The two errors can offset or compound. For a growing business closing in a seasonal trough they partly cancel; closing at a peak they add, and the peg is too low by $s_c + g(5.5 + d)$, all of which the buyer pays. Part (b) shows that an unbiased alternative exists with two years of history at sixteen times the variance of the twelve-month mean, and part (c) says exactly when the trade is worth making.

6 Robustness to a Single Aberrant Month

A window often contains a month that is not normal: a delayed payment run, a one-time receivable, a stock build. Diligence removes the ones it finds; the question is what an unremoved one costs. Let $x_1, \dots, x_k$ be the clean monthly values with order statistics $x_{(1)} \le \cdots \le x_{(k)}$, and let the contaminated sample replace one value x_i by $x_i + \Delta$.

Proposition 6.1 (Shift of the peg from one distorted month)**.** *Let $\Delta > 0$ (the case $\Delta < 0$ follows by negating all values).*

(a) The mean shifts by exactly Δ/k.

(b) For $k \ge 3$, the trimmed mean $\frac{1}{k-2}\sum_{j=2}^{k-1} x_{(j)}$ shifts by an amount in $\big[0,\ \min\{\Delta,\, x_{(k)} - x_{(2)}\}/(k-2)\big]$.

(c) For odd $k = 2m+1 \ge 3$, the median shifts by an amount in $\big[0,\ \min\{\Delta,\, x_{(m+2)} - x_{(m+1)}\}\big]$. For even $k = 2m \ge 4$ it shifts by an amount in $\big[0,\ \frac{1}{2}\min\{\Delta,\, x_{(m+2)} - x_{(m)}\}\big]$.

In (b) and (c) the shift stays bounded as $\Delta \to \infty$; in (a) it does not.

Proof. (a) is immediate. For (b) and (c), write $x'_{(j)}$ for the order statistics of the contaminated sample. Raising one value cannot lower any order statistic, so $x'_{(j)} \ge x_{(j)}$. For $j < k$, the $j+1$ smallest clean values are all at most $x_{(j+1)}$ and at most one of them was changed, so at least j contaminated values are at most $x_{(j+1)}$; hence $x'_{(j)} \le x_{(j+1)}$. Moreover $\sum_j (x'_{(j)} - x_{(j)}) = \Delta$ with every term non-negative, so the sum over any subset of indices lies in $[0, \Delta]$. For (b), the sum over $j = 2, \dots, k-1$ is also at most $\sum_{j=2}^{k-1}(x_{(j+1)} - x_{(j)}) = x_{(k)} - x_{(2)}$. For (c) with k odd, the single term $j = m+1$ is at most $x_{(m+2)} - x_{(m+1)}$. With k even the median is $\frac{1}{2}(x_{(m)} + x_{(m+1)})$, and the two terms sum to at most $(x_{(m+1)} - x_{(m)}) + (x_{(m+2)} - x_{(m+1)})$ and to at most Δ. □

In the language of robust statistics, founded by Huber [1964] and developed through the influence curve of Hampel [1974], the mean has an unbounded influence function and the median a bounded one. The global counterpart is the breakdown point, the smallest fraction of observations whose arbitrary replacement can carry the estimate to infinity: $1/k$ for the mean, $2/k$ for the trimmed mean above and $\lfloor (k+1)/2 \rfloor / k$ for the median, with limits 0, 0 and $\frac{1}{2}$. Robustness is paid for in efficiency. Under normal noise the sample median is asymptotically normal with variance $\pi\sigma^2/(2k)$, so its asymptotic relative efficiency against the mean is $2/\pi \approx 0.637$ and its standard error about $\sqrt{\pi/2} \approx 1.25$ times larger. This is the

classical large-sample result; the finite-sample figure for $k = 3$, 6 or 12 differs somewhat and I do not rely on it.

Remark 6.2 (The median is not seasonally neutral)**.** With seasonality the median of a twelve-month window estimates the median, not the mean, of the seasonal profile: in the noise-free limit it equals $\mu + \mathrm{med}(s_j)$, and the example of Section 10 has $\mathrm{med}(s_j) = -15$. A robust peg needs the seasonal adjustment first and the robust average second.

7 Neutrality of the Mechanism to Timing

A seller can change closing working capital at will by moving the timing of cash: delay a supplier payment by x and payables rise by x, so working capital falls by x, while cash rises by x. If the formula is assembled correctly the two effects cancel. The following makes the condition exact.

Index the balance-sheet items other than equity by a finite set I and let a_i be the signed book value of item i, positive for assets and negative for liabilities. The contract specifies three sets of items $C, D, N \subseteq I$ and defines $\mathrm{Cash} = \sum_{i \in C} a_i$, $\mathrm{Debt} = -\sum_{i \in D} a_i$ and $W = \sum_{i \in N} a_i$. Let $w_i = \mathbf{1}_C(i) + \mathbf{1}_D(i) + \mathbf{1}_N(i)$ be the *coverage count* of item i. Then (1) reads

$$\mathrm{Eq} = \mathrm{EV} - P + \sum_{i \in I} w_i \, a_i. \tag{8}$$

A *timing action* is a vector $\delta \in \mathbb{R}^I$ of changes to closing balances with $\sum_i \delta_i = 0$: it moves value between items without changing book equity. I assume EV and P do not depend on the action, which is the case once they are fixed in the agreement.

Theorem 7.1 (Neutrality and leakage)**.** *A timing action δ changes the equity price by*

$$\Delta\mathrm{Eq} = \sum_{i \in I} w_i \, \delta_i = \sum_{i \in I} (w_i - 1) \, \delta_i. \tag{9}$$

For a set of items $S \subseteq I$ with at least two elements, the price is unchanged by every timing action supported on S if and only if w is constant on S. In particular, if cash at bank belongs to S and is counted once, neutrality on S holds if and only if every item of S lies in exactly one of C, D, N.

Proof. The first equality in (9) is (8) applied before and after the action; the second subtracts $\sum_i \delta_i = 0$. If $w \equiv \kappa$ on S and δ is supported on S, then $\Delta\mathrm{Eq} = \kappa \sum_i \delta_i = 0$. Conversely, for distinct $i, j \in S$ the action with $\delta_i = 1$, $\delta_j = -1$ and all other entries zero is a timing action supported on S with $\Delta\mathrm{Eq} = w_i - w_j$, so neutrality forces $w_i = w_j$. □

The delayed payable is the case $\delta_{\text{cash}} = +x$, $\delta_{\text{payables}} = -x$. If cash is in C and trade payables are in N, both have $w = 1$ and the price does not move: the working-capital adjustment takes back exactly what the cash adjustment gave. Now suppose the stretched liability lies in none of the three definitions, for example a category of accrued liabilities carved out of working capital and not listed as debt. Then $w = 0$ for that item and (9) gives $\Delta\mathrm{Eq} = (0 - 1)(-x) = +x$: the buyer pays x more for an unchanged business and inherits the liability. An item in two definitions, such as an overdraft counted both as debt and

within working capital, has $w = 2$ and leaks the other way: drawing it by x to raise cash lowers the price by x.

This is, I think, the mathematical reason that definitional disputes dominate working-capital negotiations. Under exact single coverage the timing of cash is irrelevant and the only thing left to argue about is the peg. Every gap or overlap creates an item whose closing balance moves the price one for one, and which the party in control of the business between signing and closing can move. The double-entry structure behind (9) is as old as Pacioli [1494] and was formalised by Ijiri [1967] and Ellerman [1985]; the practical difficulty is that the three definitions are drafted separately. Accounting standards define cash and cash equivalents for the cash flow statement [International Accounting Standards Board, 2016, Financial Accounting Standards Board, 2016], but a purchase agreement is free to define them differently.

Remark 7.2 (Consistency of the peg with the definition)**.** The same logic applies between the peg and the closing statement. If an item is in closing working capital but was excluded from the monthly balances behind the peg, or the reverse, its entire closing balance, not a timing difference, enters the adjustment. Estimator and estimand must share one definition.

8 Collars

Many agreements suspend the adjustment inside a band around the peg. With a symmetric collar of half-width $c_0 \geq 0$ operating as a deductible, the payment for a raw difference $x = W_{T+d} - P$ is

$$a(x) = (x - c_0)^+ - (-x - c_0)^+, \tag{10}$$

which is zero for $|x| \leq c_0$, equals $x - c_0$ above the band and $x + c_0$ below it. Let Φ and φ denote the standard normal distribution function and density.

Proposition 8.1 (Expected adjustment under a collar)**.** *Let $X \sim N(b, v^2)$ with $v > 0$. Then $h(b) = \mathbb{E}[a(X)]$ is*

$$h(b) = (b - c_0)\,\Phi\Big(\frac{b - c_0}{v}\Big) + (b + c_0)\,\Phi\Big(-\frac{b + c_0}{v}\Big) + v\Big[\varphi\Big(\frac{b - c_0}{v}\Big) - \varphi\Big(\frac{b + c_0}{v}\Big)\Big]. \tag{11}$$

Moreover h is odd, $h'(b) = \Phi(\frac{b-c_0}{v}) + \Phi(-\frac{b+c_0}{v}) = \Pr(|X| > c_0) \in (0, 1]$, and consequently $h(b) = 0$ if and only if $b = 0$. If $c_0 > 0$ then $|h(b)| < |b|$ for every $b \neq 0$.

Proof. For $Y \sim N(m, v^2)$, $\mathbb{E}[Y^+] = \int_{-m/v}^{\infty} (m + vz)\varphi(z)\,dz = m\,\Phi(m/v) + v\,\varphi(m/v)$, using $\int_u^{\infty} z\varphi(z)\,dz = \varphi(u)$ and the symmetry of φ. Apply this to $Y = X - c_0$ with $m = b - c_0$ and to $Y = -X - c_0$ with $m = -b - c_0$, and subtract; since φ is even this is (11). Replacing b by $-b$ in (11) exchanges the two Φ terms with a change of sign and reverses the bracket, so $h(-b) = -h(b)$, and in particular $h(0) = 0$. For the derivative use $\varphi'(z) = -z\varphi(z)$: $\frac{d}{db}\big[(b - c_0)\Phi(\frac{b-c_0}{v}) + v\varphi(\frac{b-c_0}{v})\big] = \Phi(\frac{b-c_0}{v})$, the two density terms cancelling, and in the same way $\frac{d}{db}\big[(b + c_0)\Phi(-\frac{b+c_0}{v}) - v\varphi(\frac{b+c_0}{v})\big] = \Phi(-\frac{b+c_0}{v})$. The sum is $\Pr(X > c_0) + \Pr(X < -c_0)$, strictly positive, so h is strictly increasing and vanishes only at 0. If $c_0 > 0$ then $h' < 1$ everywhere, so $|h(b)| = |\int_0^b h'| < |b|$. □

A collar therefore does not remove a biased peg's expected transfer. It attenuates it, and for a small bias the pass-through rate is $h'(0) = 2\Phi(-c_0/v)$: with a half-width of one standard deviation of the raw difference, about 32 per cent of a small bias still changes hands in expectation. Here $b = \mathbb{E}[W_{T+d} - P]$ and v^2 is the variance of the raw difference, which for a trailing k-month peg under (IID) is $\sigma^2(1 + 1/k)$. As an illustration, $b = -120$, $v = 100$ and $c_0 = 50$ give $h(b) \approx -82.5$.

9 An Operating Identity for Level and Trend

The drift g in Section 5 has an operating interpretation. Define, for a period with revenue R and cost of goods sold K, the usual days ratios from the period-end balances: $\mathrm{DSO} = 365 \cdot \mathrm{AR}/R$, $\mathrm{DIO} = 365 \cdot \mathrm{Inv}/K$ and $\mathrm{DPO} = 365 \cdot \mathrm{AP}/K$. Then trade working capital $\mathrm{TWC} = \mathrm{AR} + \mathrm{Inv} - \mathrm{AP}$ satisfies

$$\mathrm{TWC} = R \cdot \frac{\mathrm{DSO}}{365} + K \cdot \frac{\mathrm{DIO}}{365} - K \cdot \frac{\mathrm{DPO}}{365}, \tag{12}$$

which is an identity given the definitions. It becomes a model, and an approximation, when the days are treated as stable parameters. If the days and the gross margin q are constant, so that $K = (1-q)R$, substitution gives $\mathrm{TWC} = \omega R$ with $\omega = [\mathrm{DSO} + (1-q)(\mathrm{DIO} - \mathrm{DPO})]/365$, and a change ΔR in annualised revenue requires incremental working capital $\Delta\mathrm{TWC} = \omega\,\Delta R$.

The assumptions are strong: fixed ratios, a fixed mix of customers and suppliers, and no account of the accruals and prepayments that a contractual definition adds. Within those limits the identity supplies a prior for the trend: if annualised revenue rises by ρ_R per month, then $g \approx \omega\rho_R$. For an illustrative business with $R = 7{,}300$, $q = 0.4$, $\mathrm{DSO} = 45$, $\mathrm{DIO} = 60$ and $\mathrm{DPO} = 30$ (USD thousands), $\mathrm{TWC} = 900 + 720 - 360 = 1{,}260$ and $\omega = 63/365$. Revenue growth of 73 per month in annualised terms gives $g = 73 \times 63/365 = 12.6$, and by Proposition 5.3(a) a twelve-month peg with $d = 3$ then lags the closing level by $8.5 \times 12.6 = 107.1$ before any seasonal effect.

10 An Illustrative Worked Example

The following example is illustrative. The numbers are constructed, not observed, and the noise is set to zero so that every figure isolates a bias and can be checked by hand. Amounts are in USD thousands, the level is $\mu = 1{,}000$, and the seasonal components are in Table 1. The peg is fixed on the December balance sheet and I consider two closing dates: the end of March ($d = 3$, a trough, $\tau_c = 880$) and, for contrast, the end of the following October ($d = 10$, a peak, $\tau_c = 1{,}150$).

Table 1: Illustrative monthly net working capital, $W_t = \mu + s_{m(t)}$, USD thousands.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
s_j	−60	−90	−120	−90	−60	−30	0	60	120	150	90	30
W_t	940	910	880	910	940	970	1,000	1,060	1,120	1,150	1,090	1,030

Table 2: Illustrative pegs and expected transfers for the data of Table 1, USD thousands. Window bias is $B_k(T)$; the expected adjustment is $\tau_c - P = s_c - B_k(T)$ (Theorem 4.1). Positive amounts are paid by the buyer to the seller.

Peg basis	Peg	Window bias	Expected adjustment	
		$B_k(T)$	March close	October close
Trailing 3 months (Oct to Dec)	1,090	+90	−210	+60
Trailing 6 months (Jul to Dec)	1,075	+75	−195	+75
Trailing 12 months (Jan to Dec)	1,000	0	−120	+150
Aligned (prior-year closing month)	880 or 1,150	n/a	0	0

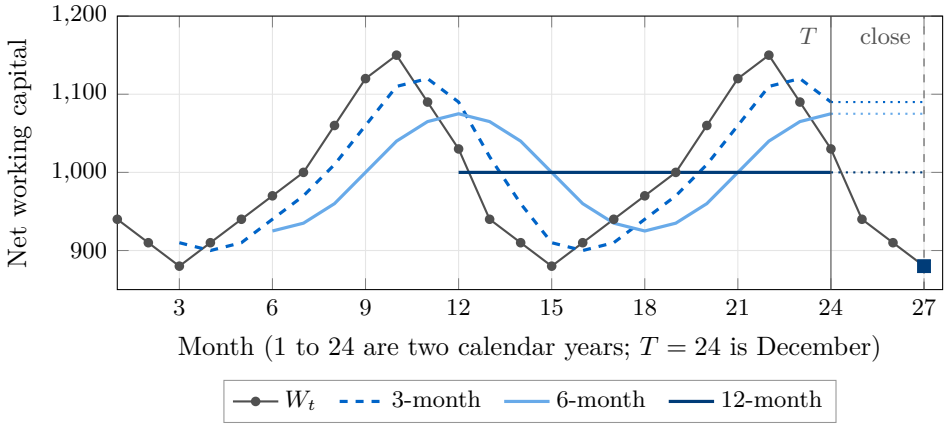


Figure 2: The illustrative series of Table 1 over two years with its rolling 3, 6 and 12 month averages. At $T = 24$ the pegs are frozen (dotted) at 1,090, 1,075 and 1,000. The square marks the normal level at a March close, 880; the drop from each dotted line to the square is the expected transfer from seller to buyer in Table 2: 210, 195 and 120.

The trailing sums are $1{,}150 + 1{,}090 + 1{,}030 = 3{,}270$ for three months, 6,450 for six months and 12,000 for twelve, giving pegs of 1,090, 1,075 and 1,000. The seasonally aligned peg with one year of history is the prior-year balance for the closing month. Table 2 reports the expected price adjustment $\tau_c - P$ under each peg; a negative figure is a reduction in price, that is, a transfer from seller to buyer.

The twelve-month peg is exactly unbiased for μ and still transfers 120 to the buyer at a March close and 150 to the seller at an October close: the transfer is s_c. The short windows are worse at the March close, because they sit in the autumn peak, and better at the October close, because they partly resemble it. No trailing window is right at both. Only the aligned peg removes the expected transfer, and with noise it would do so at the cost of the larger variance in Proposition 4.2. Figure 2 shows the series with the three rolling averages.

One distorted month. Suppose December carried a one-time item of $\Delta = 180$, so that the recorded balance is 1,210. By Proposition 6.1(a) the three pegs move by 60, 30 and 15, to 1,150, 1,105 and 1,015. The twelve-month trimmed mean moves from $(12{,}000 - 880 - 1{,}150)/10 = 997$ to $(12{,}180 - 880 - 1{,}210)/10 = 1{,}009$, a shift of 12, inside the

bound $\min\{180,\ 1{,}150 - 910\}/10 = 18$. The twelve-month median is $(970 + 1{,}000)/2 = 985$ before and after, a shift of zero, inside the bound $\frac{1}{2}\min\{180,\ 1{,}030 - 970\} = 30$. The median's clean value of 985 rather than 1,000 is the skew effect of Remark 6.2.

11 Implications for Practice

The results suggest a short discipline. Estimate the target for the closing month, not for the year. Report each trailing average with its window seasonal mean. Compare σ^2/g^2 with the thresholds of Corollary 5.2 before preferring a long window because it is smoother. Report a robust alternative beside the mean. Check that every item touched by ordinary cash management has coverage count one, and that the peg and the closing statement use one definition.

Finsider reports working-capital peg bases over trailing 3, 6 and 12 months. The analysis here is the reason for showing all three: they are estimators with different biases, and the differences between them are informative about seasonality and trend. The figures are computed in a deterministic core from ledger data and traced to source rows, which matters because Remark 7.2 makes a peg only as good as the consistency of the monthly balances behind it. Nothing in this paper is a claim about the accuracy of any product.

12 Falsifiable Predictions

The theorems are true of the model. Whether the model describes transactions is an empirical question this paper does not answer. I state five predictions about what a future study of purchase agreements and closing statements would find.

Prediction 1 (Seasonal sign of true-ups)**.** Among deals with a trailing twelve-month peg and a seasonal target, the sign of the post-closing adjustment agrees with the sign of the closing month's seasonal component more often than chance. *Refuted if* in a sample of at least 200 such deals the agreement rate is not significantly above one half.

Prediction 2 (Magnitude scales with seasonal amplitude)**.** The absolute adjustment, scaled by average working capital, increases with the target's seasonal amplitude when the peg is a trailing average, and does not when the peg is seasonally aligned. *Refuted if* the relation is indistinguishable across the two groups.

Prediction 3 (Growth favours the seller under long windows)**.** For targets with growing revenue, adjustments under twelve-month pegs are on average positive (paid to the seller), and their mean rises with the product of monthly working-capital drift and the lag $5.5 + d$. *Refuted if* that mean is zero or negative after controlling for the closing month.

Prediction 4 (Disputes follow coverage, not arithmetic)**.** In post-closing working-capital disputes, most disputed dollars concern whether an item belongs in cash, debt or working capital, or its measurement basis, and a minority concern the computation of the average. *Refuted if* a systematic coding of disputes attributes most disputed dollars to the averaging computation.

Prediction 5 (Robust pegs are closer after an abnormal month)**.** Where diligence later identifies a single non-recurring month inside the peg window, a median or trimmed-mean peg from the same window lies closer to the subsequently realised same-month level than the mean-based peg. *Refuted if* mean-based pegs are at least as close. I hold this one least firmly, because Remark 6.2 shows a robust average can add seasonal bias of its own.

13 Limitations

The additive seasonal model. In a growing business the seasonal swing usually scales with the level, a multiplicative structure. Taking logarithms restores additivity only when working capital is positive, and net working capital can be negative. The qualitative conclusion (a full-year window removes the seasonal mean and aims at the wrong month) does not depend on additivity; the exact formulas do.

Stationarity and independence. The model takes μ, the seasonal components, g and σ^2 as constants. A business that changed its payment terms or product mix during the window violates this. The headline variances assume independent noise; Lemma 3.2 gives the correction, and with positive persistence the true variances exceed σ^2/k. When d is small and noise is persistent, recent months carry information about the close that an average ignores.

Bargaining. A negotiated peg reflects leverage, information asymmetry and trades against other terms, and a seller may rationally accept a biased peg in exchange for price. The paper prices the bias. It does not predict who concedes it.

Definitions and distributions. Theorem 7.1 treats the definitions as sets of items. Real agreements also specify measurement bases (reserves, cut-off, accounting policies), and a change of basis between the peg and the closing statement leaks in ways the coverage count does not capture. Proposition 8.1 and the efficiency figure $2/\pi$ assume normal noise.

No empirical validation. Every number here is derived or constructed, and the predictions of Section 12 are untested.

14 Conclusion

The working-capital peg is an estimate of an unobserved quantity, and the purchase price is linear in its error. The trailing twelve months is unbiased for the annual mean and biased for the closing month by precisely that month's seasonal component. Short windows trade seasonal bias against trend lag in a way that can be computed. A collar attenuates a biased peg's expected transfer and never removes it. And the mechanism is neutral to the timing of cash exactly when the contractual definitions cover each affected item once, which is why the definitions, and not the arithmetic, are where the disputes are. None of this tells the parties what to agree. It tells them what each candidate number is expected to cost, and who is expected to pay.

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